

Register Number  
Name of the Candidate:

**M.Sc. DEGREE EXAMINATION, May 2015**

**(MATHEMATICS)**

**(SECOND YEAR)**

**240: MATHEMATICAL STATISTICS**

Time: Three hours

Maximum: 100 marks

**SECTION-A**

**(8×5=40)**

**Answer any EIGHT questions**

1. Prove the followings: i)  $P(B-A)=P(B)-P(A)$ , if  $A \subseteq B$  ii)  $P(A^c)=1-P(A)$  and iii)  $P(A^c \cap B) = P(B)-P(A \cap B)$  for any two events A and B.
2. If X is a random variable with probability mass function.  
 $P(X=K) = n C_k P^k (1-P)^{n-k}$  for  $k=0,1,2,\dots,n, 0 < p < 1$  then find  
i)  $E(X)$  ii)  $E(X^2)$  and iii)  $E(X^3)$
3. Suppose that X is geometrically distributed random variable. Find  $E(X)$ ,  $\text{Var}(X)$  and  $M(t)$  the moment generating function of X.
4. Prove that  $\text{Min } E[(X-cy-d]^2] = \sigma_1^2 (1-\rho^2)$ , where C and d are some constants ,  
 $\sigma_1^2 = \text{Var}(X)$  and  $\rho$  is the correlation coefficient between the random variables X and Y.
5. In the usual notation, show that  
$$\sigma_{1,3,4,\dots,n}^2 = \frac{A_{22}}{A_{22,11}}$$
6. Show that convergence in probability implies convergence in weak sense.
7. If  $X_1, X_2, \dots, X_n$  be a sample drawn from a population with distribution function  $F(x)$  and  $\bar{X} = \frac{1}{n} \sum_{k=1}^n X_k$ , show that  $E(\bar{X}) = \mu$  and  $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$
8. Describe two way classification in analysis of variance.
9. Define consistent estimator and sufficient estimator with example.
10. i) Define asymptotically efficient estimator.  
ii) State a necessary and sufficient condition for an estimate T to a be most efficient estimator of  $\theta$ .

**SECTION-B**

**(3×20=60)**

**Answer any THREE questions**

11. State and prove inversion theorem.
12. a) If  $X \sim N(\mu, \sigma^2)$ , find the central moments of even order.  
b) Let  $X \sim G(1, \beta)$ , Then prove that  $P\{X > t+s / x > s\} = P\{X > t\}$  for any two real numbers 't' and 's'.

13. a) Suppose  $X_n \xrightarrow{P} X$  and  $g$  is a continuous function defined on  $\mathbb{R}$ , Then show that  $g(X_n) \xrightarrow{P} g(X)$  as  $n \rightarrow \infty$
- b) State and prove Kolmogrov in equality
14. a) If  $X \sim \chi^2(n)$ , then prove that  $\lim_{n \rightarrow \infty} P\{\sqrt{2x} - \sqrt{2n-1} \leq z\} = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$
- b) The following table gives the yield of three varieties of wheat each grown in four plots.

Plots of land	Variety of wheat		
	Per hectare yield ('00kgs)		
	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>
1	6	5	5
2	7	5	4
3	2	3	3
4	8	7	4

Set up ANOVA table. Apply F-ratio test to check whether there is a significant difference among the average yields in the three varieties of wheat.

(Given :F at (2,9) d.f=4.26 at 5% level)

15. State and prove Cramer-Rao inequality.

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