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Register Number
Name of the Candidate:

M.Sc. DEGREE EXAMINATION, May 2015

(MATHEMATICS)

(SECOND YEAR)

230: GRAPH THEORY

Time: Three hours Maximum: 100 marks

SECTION-A **(8×5=40)**

Answer any EIGHT questions

1. Show that if a K-regular bipartite graph with K>0 has bipartition(X,Y) then $|X|=|Y|$
2. Show that if G is a tree, then $\in=\gamma-1$.
3. Show that any two edges of a block G with $\gamma=3$ lie on a common cycle.
4. Show that C(G) is well defined.
5. Show that if G is k-regular bipartite graph with k>0 then G has a perfect matching.
6. Show that $\chi'(K_{2n})=2n-1$
7. Show that $\alpha+\beta=\gamma$
8. Show that no vertex cut of a critical graph is a clique.
9. Show that if G is a connected plane graph, then $\nu-\in+\varphi=2$
10. Show that every tournament has a directed Hamilton path.

SECTION-B **(3×20=60)**

Answer any THREE questions

11. a) Show that a graph is bipartite if and only if it contains no odd cycle.
    b) Show that an edge e of G is a cut edge of G if and only if e is contained in no cycle of G.
12. a) Show that $K\leq K'\leq\delta$.
    b) Show that a non empty connected graph is enterian if and only if it has no vertices of odd degree.
13. Show that G has perfect matching if and only if $o(G-S)\leq|S|$ for all $S\subset V$.
14. a) Show that for any two integers k, $\ell\geq 2$,
    $$r(k,\ \ell)\leq r(k,\ell-1)+r(k-1,\ell)$$
    b) Show that if G is simple, then $\pi_k(G)=\pi_k(G-e)-\pi_k(G.e)$ for any edge e of G.
15. a) Show that every planar graph is 5-vertex-colorable.
    b) Show that each vertex of a disconnected tournament D with $\gamma=3$ is contained in a directed k-cycle, $3\leq k\leq\gamma$

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