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Register Number
Name of the Candidate:

M.Sc. DEGREE EXAMINATION, May 2015

(MATHEMATICS)

(SECOND YEAR)

220: SET TOPOLOGY AND FUNCTIONAL ANALYSIS

Time: Three hours Maximum: 100 marks

SECTION-A **(8×5=40)**

Answer any EIGHT questions

1. Prove that in a metric space each open sphere is an open set.
2. State and prove Cauchy's inequality.
3. Prove that any continuous image of a compact space is compact.
4. Prove that a one to –one continuous mapping of a compact space on to Hausdorff space is a homeomorphism.
5. Prove that the space R^n and C^n are connected.
6. Let x be a compact Hausdroff space. Prove that x is totally disconnected if and only if it has an open base whose sets are closed.
7. In a Banach Space . Prove the following

   i) $| \| X \| - \| Y \| | \leq \| X - Y \|$

   ii) Addition and Scalar multiplication are jointly continuous.
8. Let M be a closed linear subspace of a normed linear space N. Let x_0 be a vector not in M. If d is the distance from x_0 to M. Show that there exists a functional f_0 in N^* such that $f_0(M)=0$, $f_0(x_0)=1$ and $||f_0||=1/d$
9. State and prove Schwarz inequality.
10. If T is an operator on Hilbert space H for which $(T_{x,x})=0$ for all x. Prove that T=0.

SECTION-B **(3×20=60)**

Answer any THREE questions

11. If f and g are continuous real (Or) complex functions defined on a topological space X. Prove that

    i) f+g , αf and fg are continuous.

    ii) Further if f,g are real then $f \wedge g$ and $f \vee g$ are continuous
12. State and prove Tychonoff's theorem.
13. State and prove Ascoli's theorem.
14. State and prove Hahu Banach theorem with necessary lemmas.
15. State and prove spectral theorem.

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