Register Number :

Name of the Candidate :

5 2 7 7

B.Sc. DEGREE EXAMINATION, 2013

(MATHEMATICS)

(THIRD YEAR)

(PART - III : GROUP : A - MAIN)

(PAPER - IV)

710. VECTOR CALCULUS AND LINEAR ALGEBRA

May] [Time : 3 Hours

Maximum : 100 Marks

Answer any FIVE questions.
ALL questions carry EQUAL marks.

1. (a) Find ϕ if

$$\nabla\phi = \left(6xy + z^3\right)\hat{i} + \left(3x^2 - z\right)\hat{j} + (3xz^2 - y)\hat{k}.$$

(7)

Turn Over

(b) Find div $\vec{F}$ if

$$F = xy^2\,\hat{i} + 2x^2y\,\hat{j} - 3yz^2\,\hat{k}. \qquad (6)$$

(c) Find the directional derivative of

$$x^3 + y^3 + z^3$$

at the point (1, –1, 2) in the direction of the vector $\hat{i} + 2\hat{j} + \hat{k}$. (7)

2. (a) Show that

$$\vec{F} = (y^2 - z^2 + 3yz - 2x)\,\hat{i} + (3xz + 2xy)\,\hat{j} + (3xy - 2xz + 2z)\,\hat{k}$$

is irrotational and solenoidal. (10)

(b) If $\vec{r}$ is the position vector of the point P(x, y, z), then prove that

$$\text{grad}\left(r^n\right) = n r^{n-2}\,\vec{r}.$$

(10)

(b) Prove that is linearly dependent set of Vectors if and only if there exists a Vector in S which is linear combination of remaining vectors. (10)

8. (a) If A and B are subspaces of a vector space V, then prove that $A \cap B$ is a subspace of V and it is the largest subspace of V which is containes in A and B. (10)

(b) Define vector space V over a field F. Prove that a non-empty sbuset W of V is a subspace of V if only if N is closed with respect to Vector addition and Scalar multiplication. (10)

9. (a) Let V be a finite dimensional Vector space of dimension n. If $v_1, v_2, v_3 \ldots\ldots v_m$ are linearly independent, then prove that $m \leq n$. (10)

(b) Show that the Vectors (1, 1, 0), (0, 1, 1), (1, 0, 1) form a basis for $\Re^3$. (10)

10. (a) If V is a finite dimensional Vector Space over F and W is a subspace of V, then prove that

dim W ≤ dimV

$$\dim \frac{V}{W} = \dim V - \dim W.$$

(10)

3. (a) If

$$\vec{F} = yz\,\hat{i} + zx\,\hat{j} - xy\,\hat{k},$$

find $\int_c \vec{F}\cdot d\vec{r}$

where C is given by $x = t$; $y = t^2$; $z = t^3$ from P (0, 0, 0) to a (2, 4,8). (8)

(b) Verify Stoke's Theorem for

$$\vec{F} = x^2\,\hat{i} + xy\,\hat{j}$$

taken around the square in xy-plane whose sides are x = 0, x = a,

y = 0, y = a. (12)

4. (a) Prove that

$$\begin{vmatrix} 1 & 1 & 1 \\ b+c & c+a & a+b \\ b^2+c^2 & c^2+a^2 & a^2+b^2 \end{vmatrix} = (a-b)(b-c)c-a).$$

(10)

Turn Over

(b) Prove that

$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)$$

(10)

5. (a) Express

$$A=\begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix}$$

as the sum of a symmetric and skew symmetric matrices. (10)

(b) Show that

$$\begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

is orthogonal. (10)

6. (a) Verify Cayley-Hamilton's Theorem for the matrix

$$A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$ (13)

(b) Find the inverse of the matrix

[illegible]

(7)

7. (a) Show that the equations

x + y + z = 6.

x + 2y + 3z = 14.

x + 4y + 7z

are consistent and solve them. (10)

(b) Find the eigen values and eigen vectors of

[illegible]

(10)

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