

(b) Find the equation of the right circular cone with vertex (1, -2, -1), semi-vertical angle 60° and line

as its axis. (10)

(MATHEMATICS)

(SECOND YEAR)

(PART - III : GROUP : A - MAIN)

(PAPER - III)

650. ALGEBRA AND SOLID GEOMETRY

[Time : 3 Hours

May]
$$\frac{x-1}{z+1} = \frac{y+2}{z+1} = \frac{-4}{5} = \frac{3}{5}$$

Maximum : 100 Marks

Answer any FIVE questions.
ALL questions carry EQUAL marks.

1. (a) Solve ;

$$2x^6 - 3x^5 + 5x^4 + 6x^3 - 27x + 81 = 0$$

if one root is $\sqrt{2}+i$. (10)

Turn Over

Register Number :

Name of the Candidate :

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B.Sc. DEGREE EXAMINATION, 2013

(b) Solve the equation

$$8x^4 - 90x^3 + 315x^2 - 405x + 162 = 0$$

given that the roots are in geometric progression. (10)

2. (a) If $\alpha, \beta, \gamma, \delta$ are the roots of

$$x^4 + px^3 + qx^2 + rx + s = 0,$$

then find

and (10)

(b) Solve :

$$3x^6 + x^5 - 27x^4 + 27x^2 - x - 3 = 0. \quad (10)$$

3. (a) State and prove Fermat's Theorem. (8)

(b) Prove that $n^5 - n$ is divisible by 5, for all $n \geq 2$. (6)

(c) Prove that $3^{4n+2} + 5^{2n+1}$ is divisible by 14. (6)

4. (a) If $f: A \rightarrow B$ and $g: B \rightarrow C$, are bijections, then prove that

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$$

and $(f^{-1})^{-1} = f$. (10)

(b) Prove that the lines

are coplanar and find equation of the plane containing them. (10)

9. (a) Find the equation of a sphere which touches through

$$x - 2y - 2z - 7 = 0$$

at $(3, -1, -1)$ and passes through $(1, 1, -3)$. (10)

(b) Find the equation of the sphere whose centre is $(1, 4, 2)$ with radius 3 units. (10)

10. (a) Find the equation of a cone whose vertex is $(1, 1, 0)$ and whose guiding curve is

$$y = 0;$$

$$x^2 + z^2 = 4. \quad (10)$$

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7. (a) A line makes angles $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube. Prove that

$$(6)$$

- (b) Find the equation of the plane passing through $(2, 2, 1)$ and $(9, 3, 6)$ and perpendicular to the plane

$$2x + 6y + 6z = 9. \quad (8)$$

- (c) Find the angle between the planes

$$2x - y + z = 6$$

$$\text{and } x + y + 2z = 7. \quad (6)$$

8. (a) Find the image of the point $(1, 3, 4)$ in the plane $2x - y + z + 3 = 0$. Prove that image of the line

$$\frac{x+3}{1} = \frac{y-5}{-5} = \frac{z-2}{-10}. \quad (10)$$

- (b) If $f : x \rightarrow y$ is a function,

$$A \subseteq x \text{ and } B \subseteq y,$$

then prove that

$$(i) A \subseteq f^{-1}(f(A)).$$

$$(ii) f(f^{-1}(B)) \subseteq B.$$

Is strict inequality valid in (i) and (ii). -
Justify. (10)

5. (a) Verify that union of two subgroups of group is also a sub group. (8)

- (b) State and prove fundamental theorem on homomorphism of groups. (12)

6. (a) Prove that every finite integral domain is a field. (10)

- (b) Prove that the set of all real numbers of the form $a + b\sqrt{2}$ where a and b are rational is a ring with usual addition and multiplication. (10)