Register Number :

Name of the Candidate :

5 2 7 2

B.Sc. DEGREE EXAMINATION, 2013

(MATHEMATICS)

(SECOND YEAR)

(PART - III : GROUP : A - MAIN)

(PAPER - II)

640. ANALYSIS - II

May] [Time : 3 Hours

Maximum : 100 Marks

Answer any FIVE questions.

ALL questions carry EQUAL marks.

1. (a) Evaluate :

$$\int \frac{3x+1}{(x-1)^2(x+3)}\,dx. \qquad (7)$$

(b) Evaluate :

$$\int \frac{6x+5}{\sqrt{6+x-2x^2}}\,dx. \qquad (7)$$

Turn Over

(c) Evaluate :

$$\int\sqrt{(x-3)(7-x)}\,dx. \qquad (6)$$

2. (a) Show that

$$\int_0^{\pi/2}\frac{(\sin x)^{3/2}}{(\sin x)^{3/2}+(\cos x)^{3/2}}\,dx=\frac{\pi}{4}. \qquad (10)$$

(b) Evaluate :

$$\int\frac{x+\sin x}{1+\cos x}\,dx. \qquad (10)$$

3. (a) If

$$u_n=\int_0^{\pi/2}x^n\sin x\,dx$$

and n is a positive integer, prove that

$$u_n+n(n-1)u_{n-2}=n\left(\frac{\pi}{2}\right)^{n-1} \qquad (10)$$

(b) Find a reduction formula for the integral

$$I_{2n+1}=\int_0^{\infty}x^{2n+1}e^{-x/2}\,dx$$

and hence, show that $I_7 = 48$. (10)

4. (a) Find the area of loop of the curve

$$y^2=x^2\frac{a+x}{a-x}. \qquad (10)$$

(b) Find the surface of the solid of revolution about the x-axis of the loop of the curve

$$x=t^2,$$

$$y=t-\frac{t^3}{3}. \qquad (10)$$

5. (a) Find the centroid of a hollow hemisphere.

(10)

(b) Find the moment of inertia of a rectangle of sides 2a and 2b about an axis through its centre of gravity parallel to the 2b edge.

(10)

Turn Over

6. (a) Find the value of

$$\iint xy\, dx\, dy$$

taken over the positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. (10)

(b) Find the volume of the paraboloid of revolution

$$x^2 + y^2 = 4z$$

cut-off by the plane $z = 4$. (10)

7. (a) Eliminate the arbitrary function from

$$z = x\, f\left(\frac{y}{x}\right) + y\phi(x).$$ (10)

(b) Solve :

$$\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}.$$ (10)

8. (a) Solve :

$$xy\,(p^2 + 1) = (x^2 + y^2)\,p.$$ (10)

(b) Find the orthogonal trajectories of the family of curvatures

$$y^2 + 3x^2 = 2ax,$$

where a is a variable parameter. (10)

9. (a) Solve :

$$(D^2 - 13D + 12)\, y = e^{-2x} + 5e^{x}.$$ (10)

(b) Solve :

$$(D^2 - 3D + 2)y = \sin 3x.$$ (10)

10. (a) Solve :

$$(mz - ny)\, p + (nx - lz)\, q = ly - mx.$$ (10)

(b) Find the singular integral of

$$z = px + qy + \sqrt{1 + p^2 + q^2}.$$ (10)